Introduction

Time-Series Data: A time series is a set of measurements, ordered over time, on a particular quantity of interest.

- Numerical data ordered over time
- The time intervals can be annually, quarterly, daily, hourly, etc.
- The sequence of the observations is important
- Example:

Year:	2008	2009	2010	2011	2012
Sales:	75.3	74.2	78.5	79.7	80.2

The Importance of Forecasting

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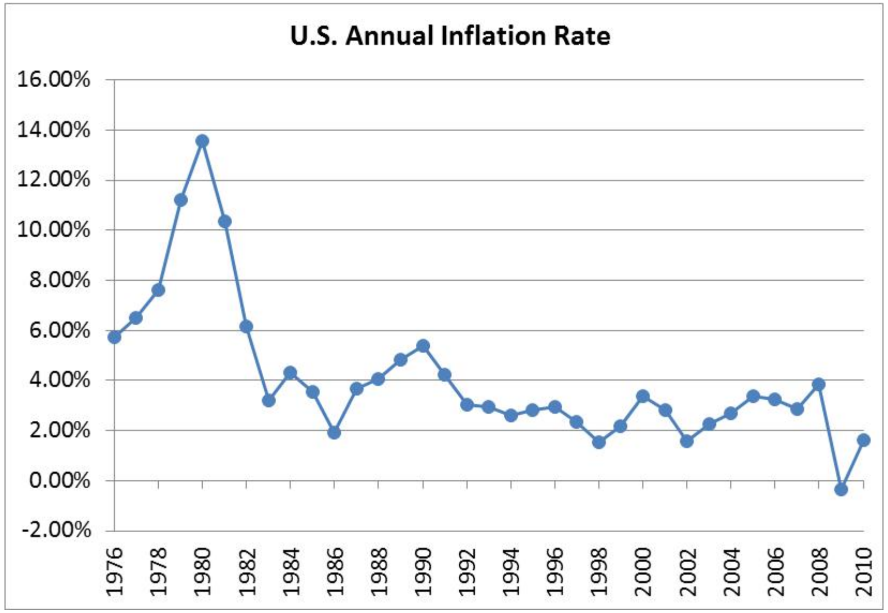
- Government needs to forecast unemployment, interest rates, expected revenues from income taxes to formulate policies
- Marketing executives need to forecast demand, sales, consumer preferences in strategic planning
- College administrators need to forecast enrollments to plan for facilities and for faculty recruitment
- Retail stores need to forecast demand to control inventory levels, hire employees and provide training

Time-Series Plot

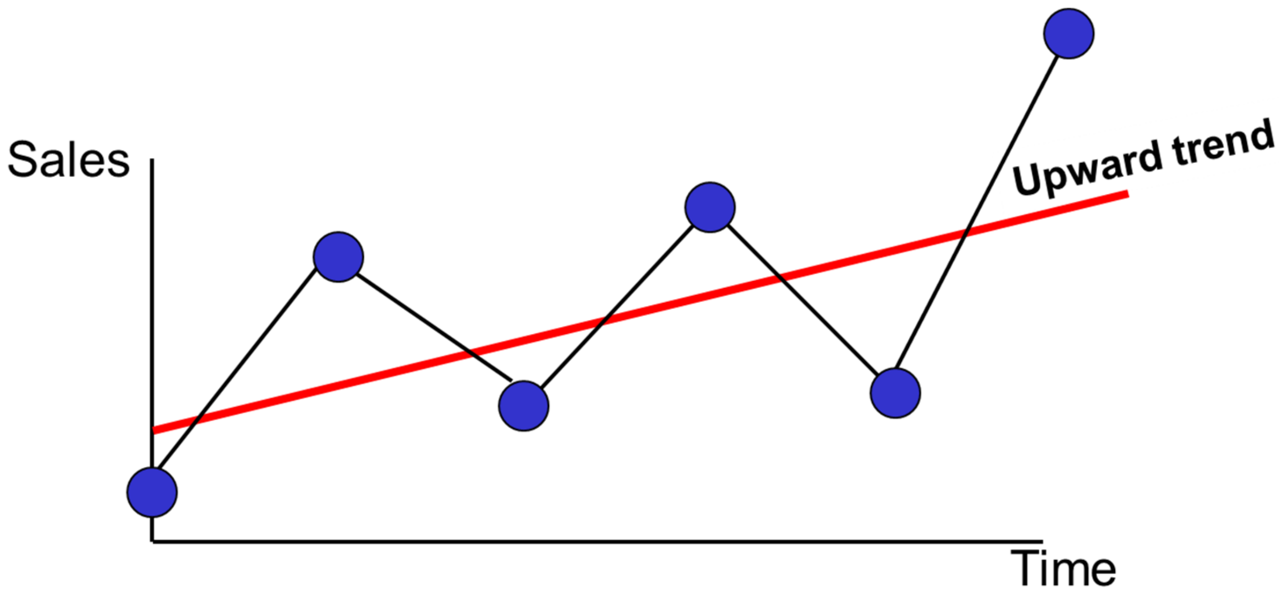
A time-series plot is a two-dimensional plot of time-series data

- the vertical axis measures the variable of interest
- the horizontal axis corresponds to the time periods



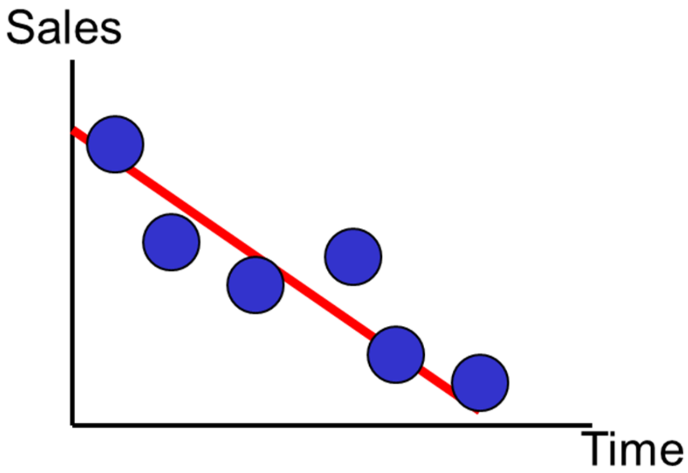
Trend Component (1 of 2)

- Long-run increase or decrease over time (overall upward or downward movement)
- Data taken over a long period of time

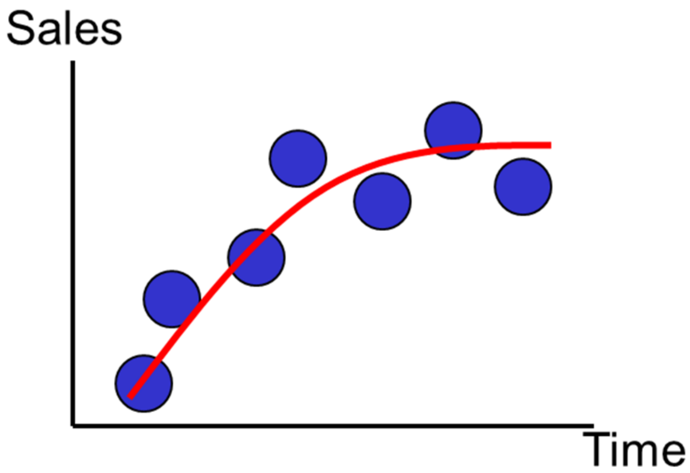


Trend Component (2 of 2)

- Trend can be upward or downward
- Trend can be linear or non-linear



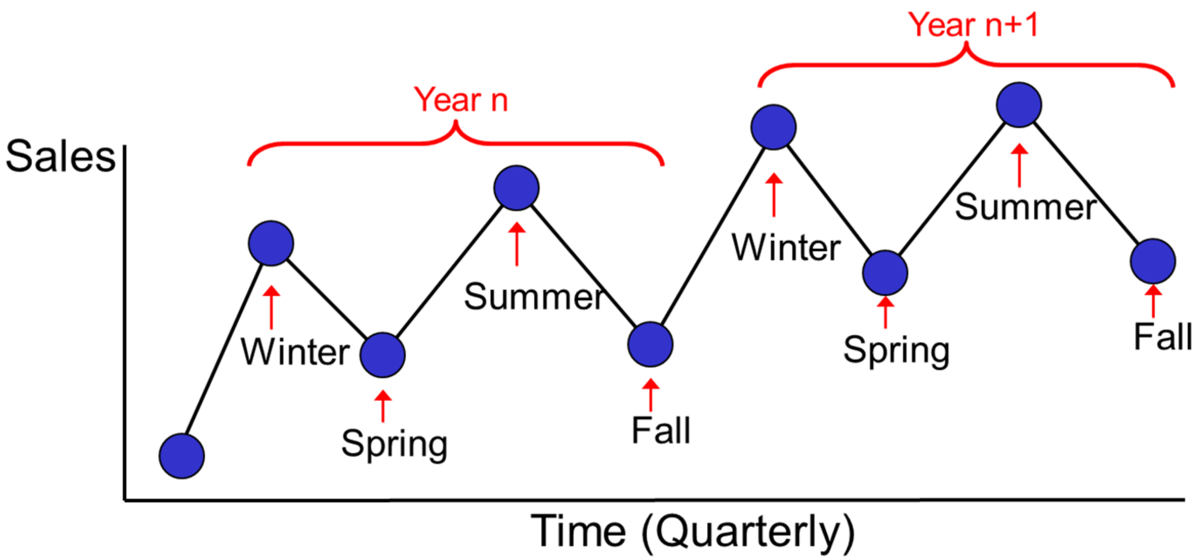
Downward linear trend



Upward nonlinear trend

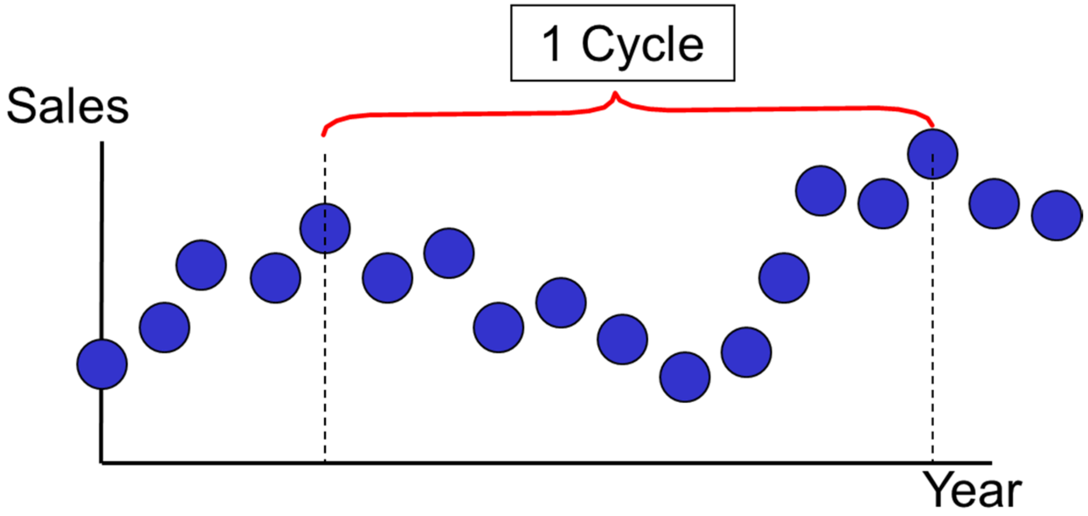
Seasonal Component

- Short-term regular wave-like patterns
- Observed within 1 year
- Often monthly or quarterly



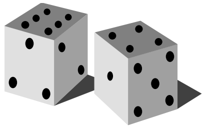
Cyclical Component

- Long-term wave-like patterns
- Regularly occur but may vary in length
- Often measured peak to peak or trough to trough



Irregular Component

- Unpredictable, random, “residual” fluctuations
- Due to random variations of
  - Nature
  - Accidents or unusual events
- “Noise” in the time series



Time Series Forecasts

Uses **time-ordered sequence** of observations at regular intervals

- *Trend* - long-term movement in data
- *Seasonality* - short-term regular variations in data
- Cycle – wavelike variations of more than one year's duration
- *Irregular variations* - caused by unusual circumstances
- *Random variations* - caused by chance

Techniques for Averaging

- Moving average
- Weighted moving average
- Exponential smoothing

Moving Averages

- *Moving average* – A technique that averages a number of recent actual values, updated as new values become available.

$$F_t = MA_n = \frac{A_{t-n} + \ldots A_{t-2} + A_{t-1}}{n}$$

- *Weighted moving average* – More recent values in a series are given more weight in computing the forecast.

$$F_t = w_1 A_{t-1} + w_2 A_{t-2} + w_3 A_{t-3} + \ldots + w_n A_{t-n}$$

Moving Average Example

Calculate a three-period moving average forecast for demand in period 6

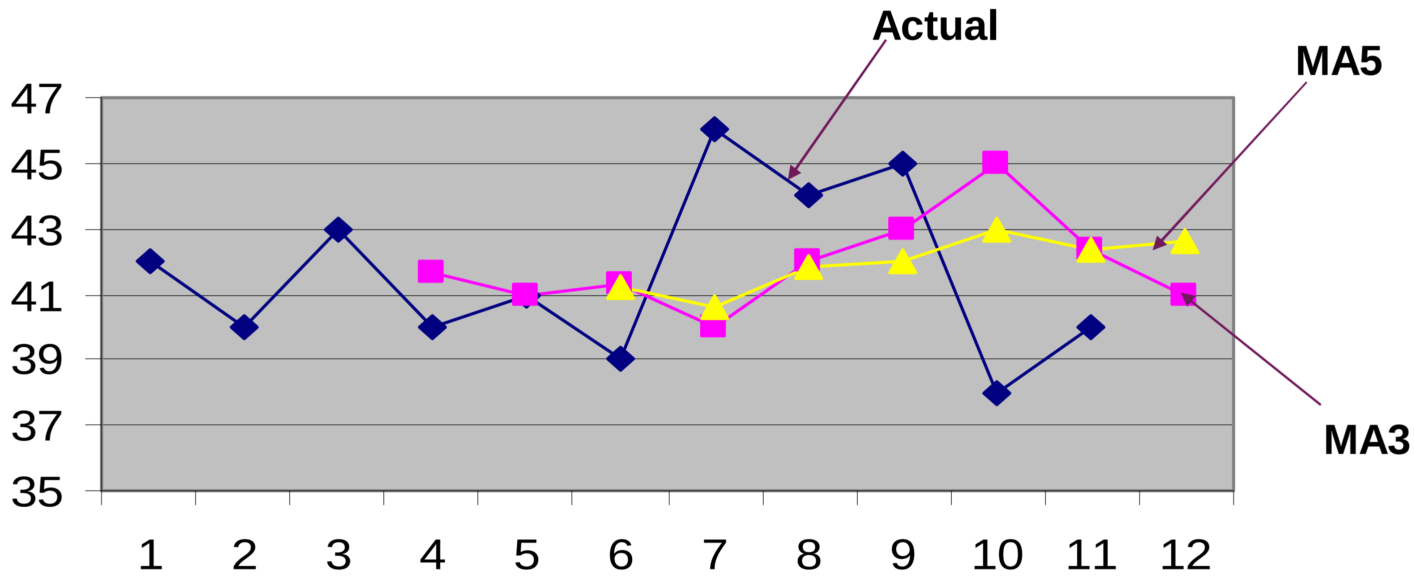
Period	Demand	
1	42	
2	40	
3	43	the 3 most recent demands
4	40	
5	41	

$$F_6 = \frac{43 + 40 + 41}{3} = 41.33$$

If the actual demand in period 6 is 38, then the moving average forecast for period 7 is:

$$F_7 = \frac{40 + 41 + 38}{3} = 39.67$$

Simple Moving Average



Takeaways:

- Fewer data points (e.g., the 3 month moving average) is more sensitive to real life and is a more dynamic forecast.
- Larger data points (e.g., the 5 month moving average) is smoother (less reactionary)

Weighted Moving Average Example

Period	Demand	Weight
1	42	
2	40	10%
3	43	20%
4	40	30%
5	41	40%

$$F_6 = .10(40) + .20(43) + .30(40) + .40(41) = 41.0$$

Takeaways:

- Choice of weights must add up to 100%
- Choice of weights are discretionary
- Choice of weights are often based on trial and error, and forecaster's experience.

Exponential Smoothing

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

- *Premise*--The most recent observations might have the highest predictive value.
  - Therefore, we should give more weight to the more recent time periods when forecasting.
  - Uses most recent period's actual and forecast data
- Weighted averaging method based on previous forecast plus a percentage of the forecast error
- *A-F* is the error term, α is the % feedback

Exponential Smoothing Example

Period (t)	Actual Demand
1	42
2	40
3	43
4	40
5	41
6	39
7	46
8	44
9	45
10	38
11	40

Exponential Smoothing Example

Period (t)	Actual Demand	$\alpha = .10$ Forecast	$\alpha = .40$ Forecast
1	42	—	—
2	40	42	42
3	43	41.8	41.2
4	40	41.92	41.92
5	41	41.73	41.15
6	39	41.66	41.09
7	46	41.39	40.25
8	44	41.85	42.55
9	45	42.07	43.13
10	38	42.35	43.88
11	40	41.92	41.53
12		41.73	40.92

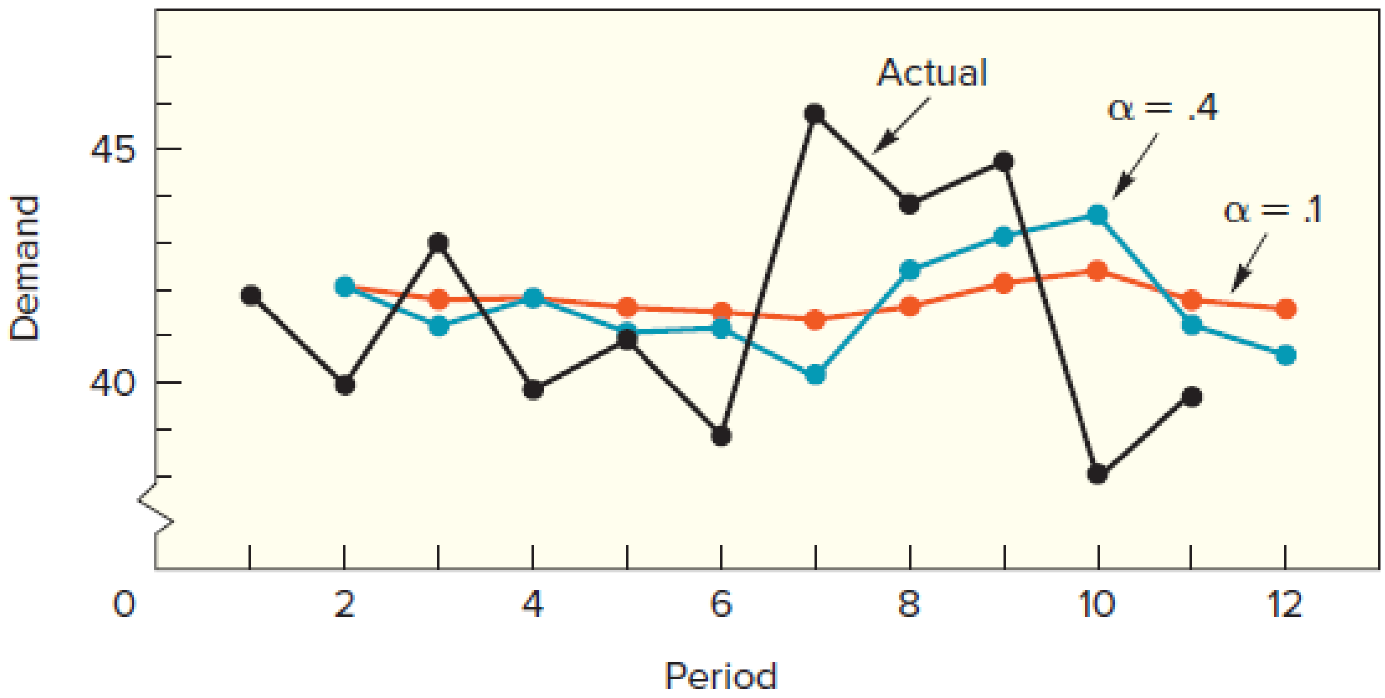
starting forecast

Period 2 forecast is a naïve forecast

$$F_3 = F_2 + \alpha(A_2 - F_2)$$
$$\alpha = 0.10, F_3 = 42 + 0.10(40 - 42) = 42 + 0.10 \times (-2) = 42 - 0.2 = 41.8$$

$$F_4 = F_3 + \alpha(A_3 - F_3)$$
$$\alpha = 0.10, F_4 = 41.8 + 0.10(43 - 41.8) = 41.8 + 0.10 \times 1.2 = 41.8 + 0.12 = 41.92$$

Picking a Smoothing Constant

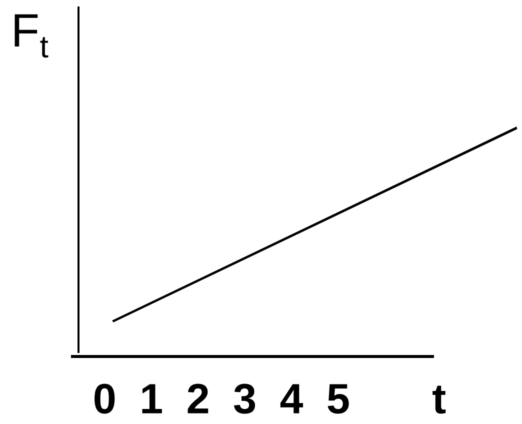


Takeaway:

- Low α, less sensitive to forecast error, relatively smoother (stable) forecast
- Large α, more sensitive to forecast error, relatively more dynamic forecast

Linear Trend Equation

$$F_t = a + bt$$



- F_t = Forecast for period t
- t = Specified number of time periods
- a = Value of F_t at t = 0
- b = Slope of the line

Calculating a and b

$$b = \frac{n\sum(ty) - \sum t \sum y}{n\sum t^2 - (\sum t)^2}$$

$$a = \frac{\sum y - b\sum t}{n}$$

Time-Series: Linear Trend Example

Period (Week)	Demand (Sales/Week)
1	150
2	157
3	162
4	166
5	177

Linear Trend Equation Example

t Week	t^2	y Sales	ty
1	1	150	150
2	4	157	314
3	9	162	486
4	16	166	664
5	25	177	885
$\Sigma t = 15$ $(\Sigma t)^2 = 225$	$\Sigma t^2 = 55$	$\Sigma y = 812$	$\Sigma ty = 2499$

Linear Trend Calculation

$$\mathbf{b} = \frac{5\,(2499) - 15(812)}{5(55) - 225} = \frac{12495 - 12180}{275 - 225} = 6.3$$

$$a = \frac{812 - 6.3(15)}{5} = \mathbf{143.5}$$

$$y = 143.5 + 6.3t$$