

Stevenson

Ch. 3

Russell & Taylor

Ch. 12

Forecasting

"Who in their right mind would ever need more than 640k of ram!?" -- Bill Gates, 1981

[Unverified source]



"JOHNSON'S OUR HEAD OF FORECASTING,
HE ALSO WORKS PART-TIME AS A SUCCESSFUL
SCIENCE FICTION WRITER."

Learning Objectives

- List the elements of a good forecast.
- Outline the steps in the forecasting process.
- Basic measure(s) of forecast accuracy.
- Compare and contrast qualitative and quantitative approaches to forecasting.
- Basic forecasting techniques: averaging, trend and seasonal, and associative.
- Evaluating and controlling forecasts.

FORECAST:

- A statement about the future value of a variable of interest such as demand.
- Forecasting is used to make *informed* decisions
 - Match supply to demand.
- Two important aspects:
 - Level of demand
 - Degree of accuracy
- Long-range – plan the system (strategic)
- Short-range – plan to use the system (on-going operations)

Forecasts

- Forecasts affect decisions and activities throughout an organization

Accounting	Cost/profit estimates
Finance	Cash flow and funding
Human Resources	Hiring/recruiting/training
Marketing	Pricing, promotion, strategy
MIS	IT/IS systems, services
Operations	Schedules, workloads
Product/service design	New products and services

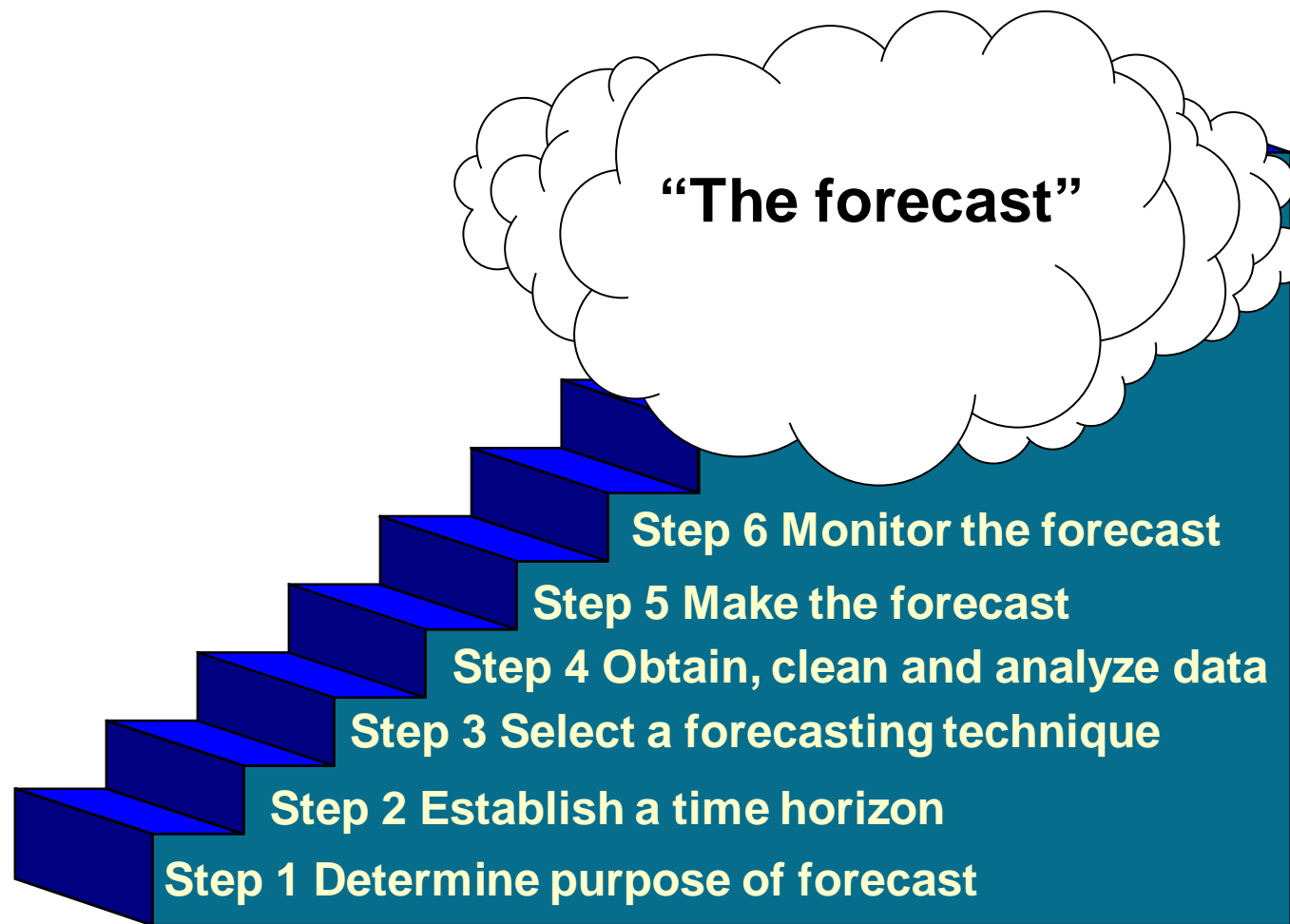
Features of Forecasts

- Assumes causal system
past ==> future
- Forecasts are rarely perfect
- Forecast accuracy decreases
as time horizon increases

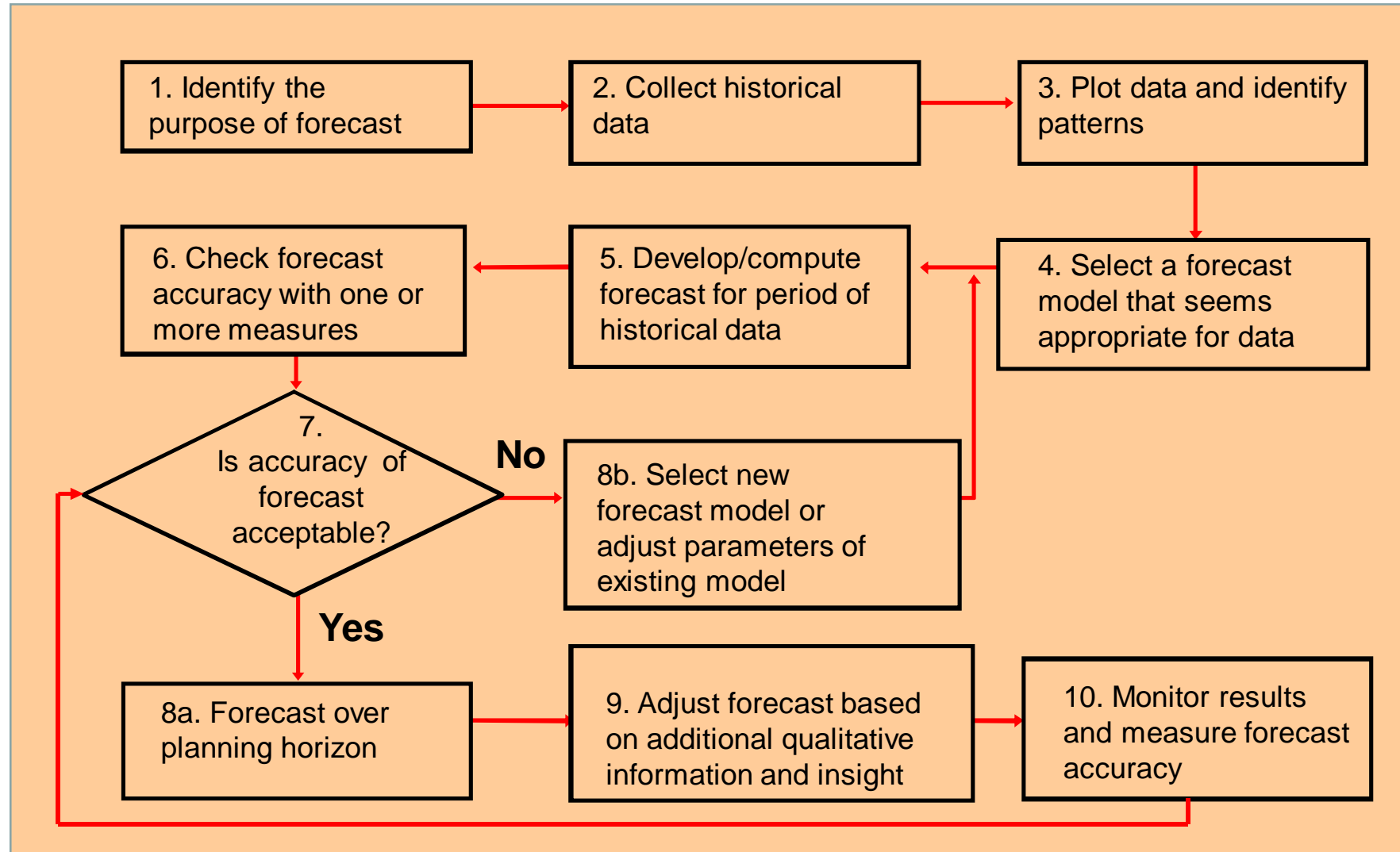
Elements of a Good Forecast



Steps in the Forecasting Process



Forecasting Process



Forecast Accuracy

- Perfect forecast (i.e., no error) is practically impossible.
- Forecaster's objective: minimize forecast error within reason.
- Practical implications:
 - Calculate error
 - Track error over time
 - Learn to improve forecasts and reduce errors over time.
- Error = difference between actual value and predicted value

$$e_t = A_t - F_t$$

e_t is error of the t^{th} period

F_t is the forecast of the t^{th} period

Forecast Accuracy

- Mean Absolute Deviation (MAD)
 - Average absolute error
 - Easiest to compute
 - Weights errors linearly.
- Mean Squared Error (MSE)
 - Average of squared error
 - Gives more weight to larger errors, which typically cause more problems
- Mean Absolute Percent Error (MAPE)
 - Average absolute percent error
 - MAPE should be used when there is a need to put errors in perspective. For example, an error of 10 in a forecast of 15 is huge. Conversely, an error of 10 in a forecast of 10,000 is insignificant. Hence, to put large errors in perspective, MAPE would be used.

MAD, MSE, and MAPE

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{forecast}|}{n}$$

$$\text{MSE} = \frac{\sum (\text{Actual} - \text{forecast})^2}{n - 1}$$

$$\text{MAPE} = \frac{[\sum |e_t| / A_t]}{n} (100)$$

Period	Actual	Forecast
1	217	215
2	213	216
3	216	215
4	210	214
5	213	211
6	219	214
7	216	217
8	212	216

Forecast Accuracy Example

Period	Actual	Forecast	(A - F)			[Error ÷ Actual] × 100
			Error	Error	Error ²	
1	217	215	2	2	4	.92%
2	213	216	-3	3	9	1.41
3	216	215	1	1	1	.46
4	210	214	-4	4	16	1.90
5	213	211	2	2	4	.94
6	219	214	5	5	25	2.28
7	216	217	-1	1	1	.46
8	212	216	-4	4	16	1.89
			<u>-2</u>	<u>22</u>	<u>76</u>	<u>10.26%</u>

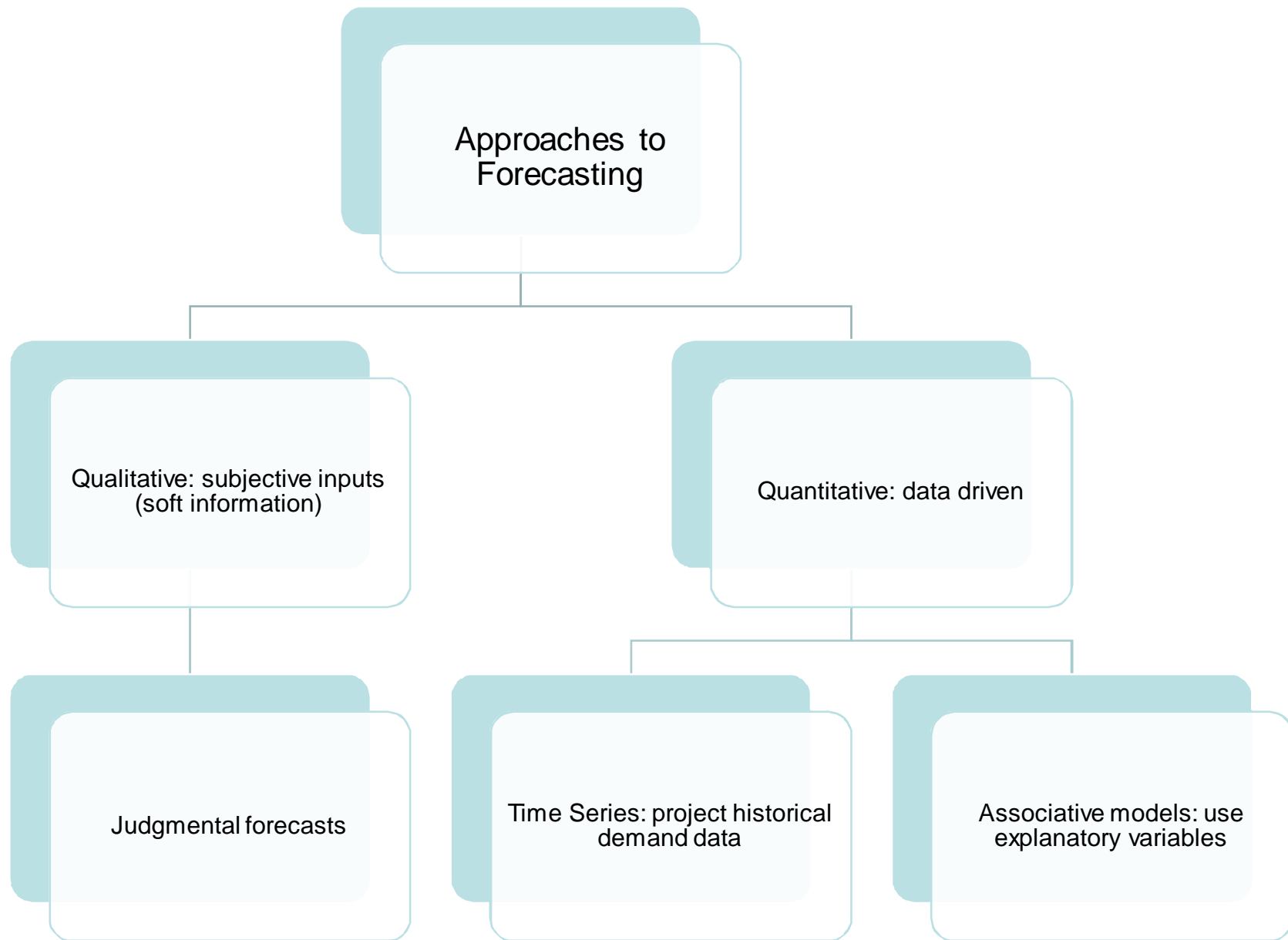
Forecast Accuracy Example

In the 8th period:

$$\text{MAD} = \frac{\sum |e|}{n} = \frac{22}{8} = 2.75$$

$$\text{MSE} = \frac{\sum e^2}{n - 1} = \frac{76}{8 - 1} = 10.86$$

$$\text{MAPE} = \frac{\sum \left[\frac{|e|}{\text{Actual}} \times 100 \right]}{n} = \frac{10.26\%}{8} = 1.28\%$$



Types of Forecasts

- Judgmental - uses subjective inputs
- Time series - uses historical data assuming the future will be like the past
- Associative models - uses explanatory variables to predict the future

Judgmental Forecasts

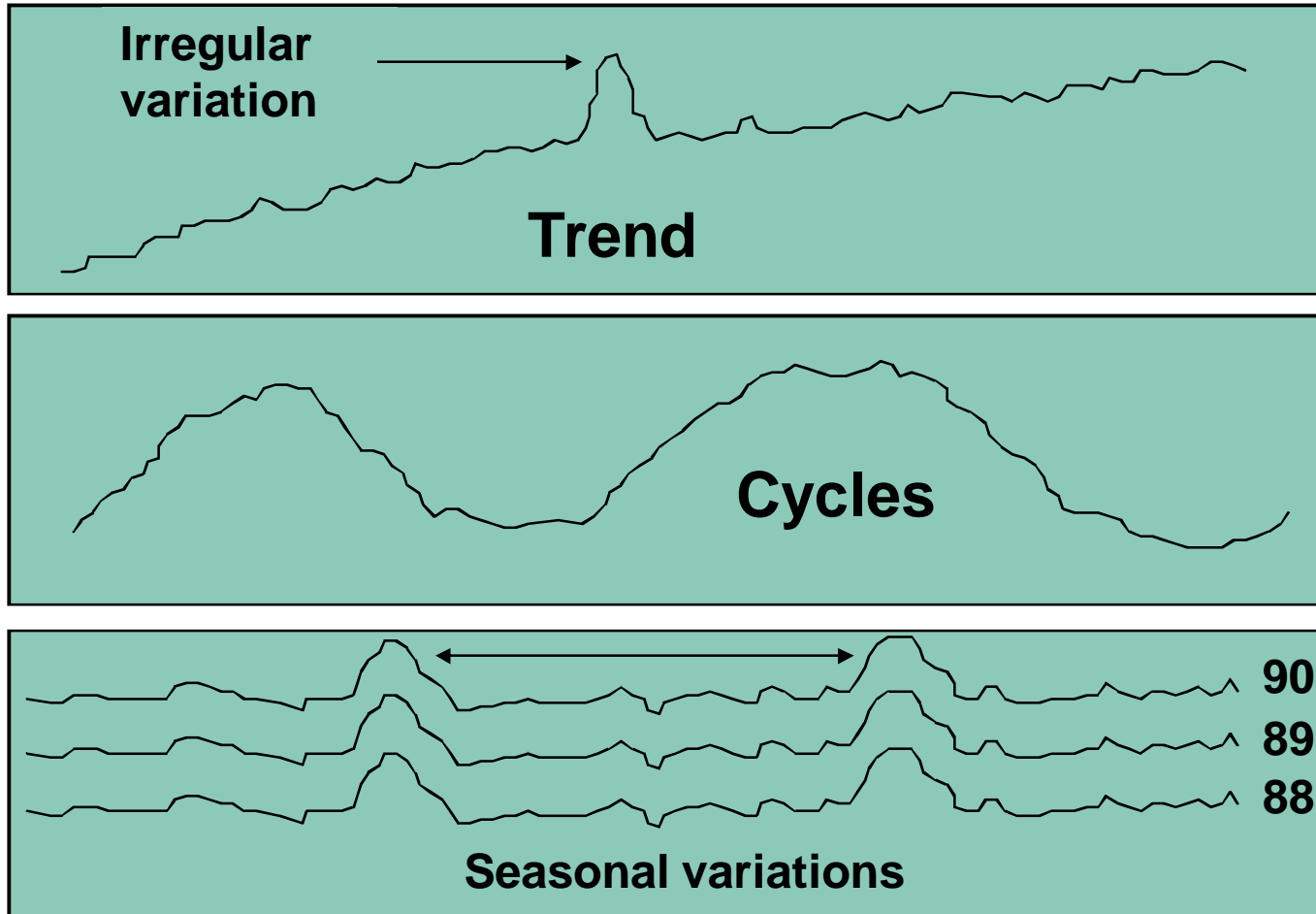
- Executive opinions
- Sales force opinions
- Consumer surveys
- Outside opinion
- Delphi method
 - Opinions of managers and staff
 - Achieves a consensus forecast

Time Series Forecasts

Uses **time-ordered sequence** of observations at regular intervals

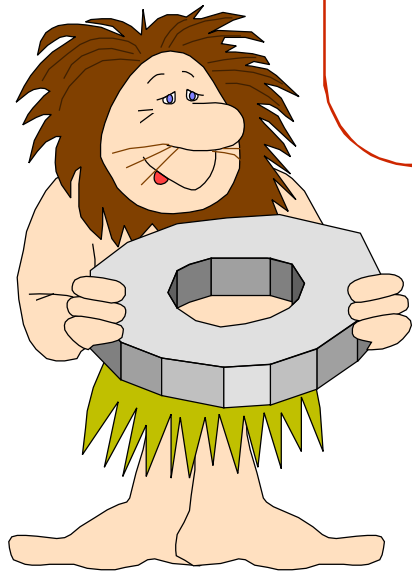
- Trend - long-term movement in data
- Seasonality - short-term regular variations in data
- Cycle – wavelike variations of more than one year’s duration
- Irregular variations - caused by unusual circumstances
- Random variations - caused by chance

Forecast Variations



Naive Forecasts

**Uh, give me a minute...
We sold 250 wheels last
week... Now, next week
we should sell...**



**The forecast for any period equals
the previous period's actual value.**

Naïve Forecasts

- Simple to use
- Virtually no cost
- Quick and easy to prepare
- Data analysis is nonexistent
- Easily understandable
- Cannot provide high accuracy
- Can be a standard for accuracy

Techniques for Averaging

- Moving average
- Weighted moving average
- Exponential smoothing

Moving Averages

- Moving average – A technique that averages a number of recent actual values, updated as new values become available.

$$F_t = MA_n = \frac{A_{t-n} + \dots + A_{t-2} + A_{t-1}}{n}$$

- Weighted moving average – More recent values in a series are given more weight in computing the forecast.

$$F_t = w_1 A_{t-1} + w_2 A_{t-2} + w_3 A_{t-3} + \dots + w_n A_{t-n}$$

Moving Average Example

Calculate a three-period moving average forecast for demand in period 6

Period	Demand
1	42
2	40
3	43
4	40
5	41

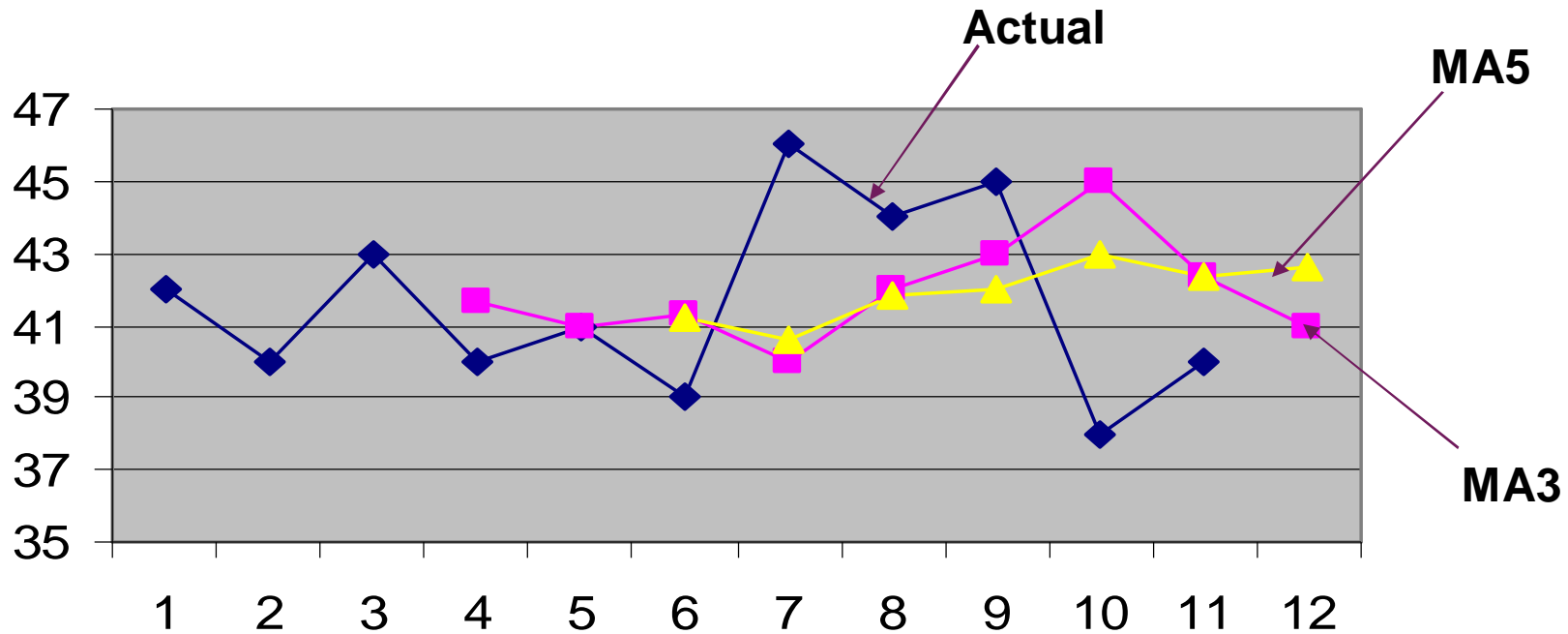
} the 3 most recent demands

$$F_6 = \frac{43 + 40 + 41}{3} = 41.33$$

If the actual demand in period 6 is 38, then the moving average forecast for period 7 is:

$$F_7 = \frac{40 + 41 + 38}{3} = 39.67$$

Simple Moving Average



Takeaways:

- Fewer data points (e.g., the 3 month moving average) is more sensitive to real life and is a more dynamic forecast.
- Larger data points (e.g., the 5 month moving average) is smoother (less reactionary)

Weighted Moving Average Example

<u>Period</u>	<u>Demand</u>	<u>Weight</u>
1	42	
2	40	10%
3	43	20%
4	40	30%
5	41	40%

$$F_6 = .10(40) + .20(43) + .30(40) + .40(41) = 41.0$$

Takeaways:

- Choice of weights must add up to 100%
- Choice of weights are discretionary
- Choice of weights are often based on trial and error, and forecaster's experience.

Exponential Smoothing

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

- *Premise*--The most recent observations might have the highest predictive value.
 - Therefore, we should give more weight to the more recent time periods when forecasting.
 - Uses most recent period's actual and forecast data
- Weighted averaging method based on previous forecast plus a percentage of the forecast error
- $A-F$ is the error term, α is the % feedback

Exponential Smoothing Example

Period (t)	Actual Demand
1	42
2	40
3	43
4	40
5	41
6	39
7	46
8	44
9	45
10	38
11	40

Exponential Smoothing Example

Period (t)	Actual Demand	$\alpha = .10$ Forecast	$\alpha = .40$ Forecast
1	42	—	—
2	40	42	42
3	43	41.8	41.2
4	40	41.92	41.92
5	41	41.73	41.15
6	39	41.66	41.09
7	46	41.39	40.25
8	44	41.85	42.55
9	45	42.07	43.13
10	38	42.35	43.88
11	40	41.92	41.53
12		41.73	40.92

Period 2
forecast
is a naïve
forecast

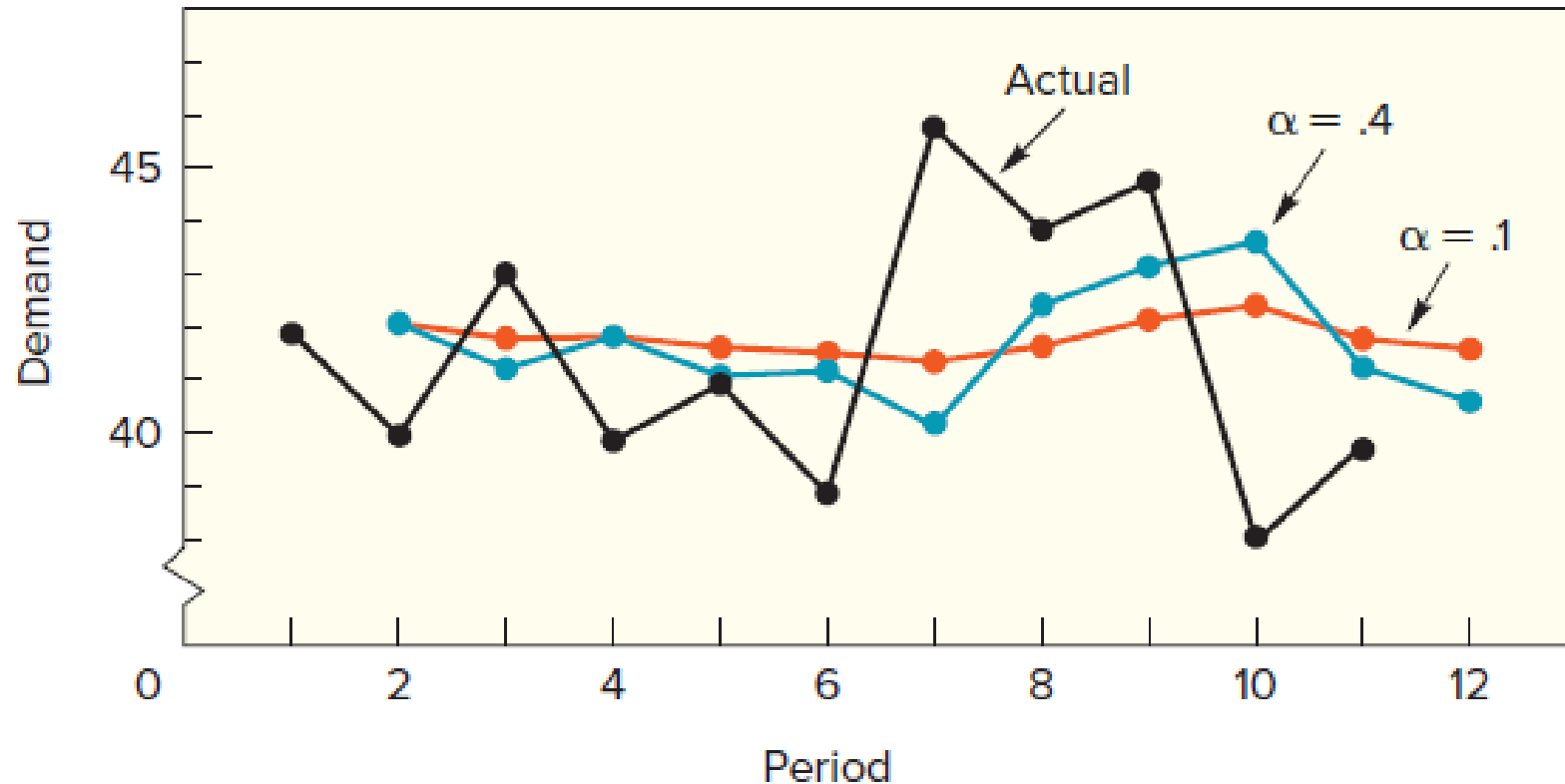
$$F_3 = F_2 + \alpha(A_2 - F_2)$$

$$\alpha = 0.10, F_3 = 42 + 0.10(40 - 42) = 42 + 0.10 \times (-2) = 42 - 0.2 = 41.8$$

$$F_4 = F_3 + \alpha(A_3 - F_3)$$

$$\alpha = 0.10, F_4 = 41.8 + 0.10(43 - 41.8) = 41.8 + 0.10 \times 1.2 = 41.8 + 0.12 = 41.92$$

Picking a Smoothing Constant



Takeaway:

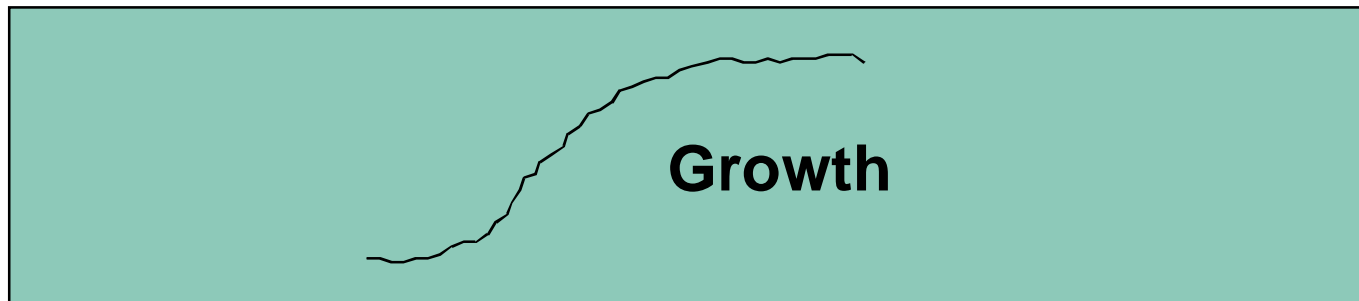
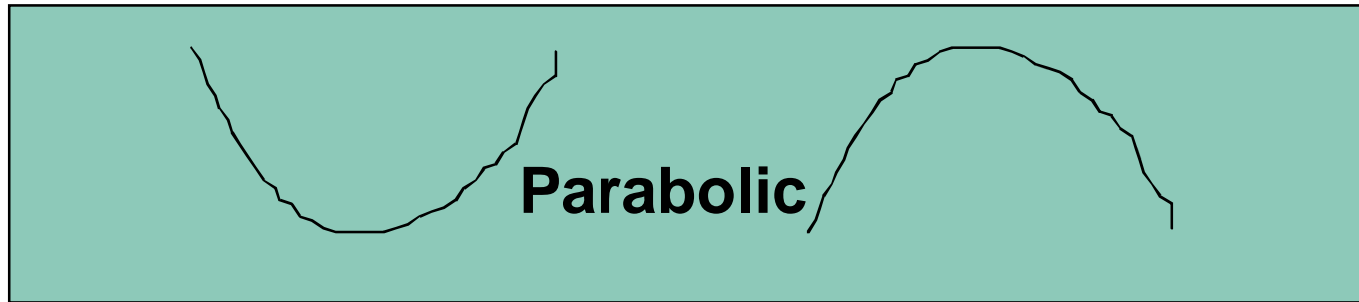
- Low α , less sensitive to forecast error, relatively smoother (stable) forecast
- Large α , more sensitive to forecast error, relatively more dynamic forecast

Period	Demand [Actual]
1	42
2	40
3	43
4	40
5	41
6	39
7	46
8	44
9	45
10	38
11	40

Compare Errors/MAD/MSE/MAPE for different techniques/variables: Periods 3 through 11

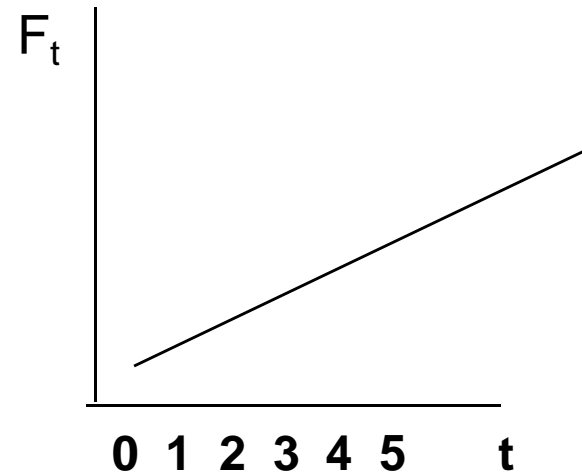
Naive Technique		Simple Moving Average Technique Forecast				Exponential Smoothing Technique					
Forecast	Error	2 Period	Error	3 Period	Error	Alpha = 10%	Error	Alpha = 20%	Error	Alpha = 30%	Error
N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
42	-2	N/A	N/A	N/A	N/A	42	-2	42	-2	42	-2
40	3	41	2	N/A	N/A	41.8	1.2	41.6	1.4	41.4	1.6
43	-3	41.5	-1.5	41.7	-1.7	41.92	-1.92	41.88	-1.88	41.88	-1.88
40	1	41.5	-0.5	41	0	41.73	-0.73	41.5	-0.5	41.32	-0.32
41	-2	40.5	-1.5	41.3	-2.3	41.66	-2.66	41.4	-2.4	41.22	-2.22
39	7	40	6	40	6	41.39	4.61	40.92	5.08	40.55	5.45
46	-2	42.5	1.5	42	2	41.85	2.15	41.94	2.06	42.19	1.81
44	1	45	0	43	2	42.07	2.93	42.35	2.65	42.73	2.27
45	-7	44.5	-6.5	45	-7	42.36	-4.36	42.88	-4.88	43.41	-5.41
38	2	41.5	-1.5	42.3	-2.3	41.92	-1.92	41.9	-1.9	41.79	-1.79
MAD	3.11		2.33				2.50		2.53		2.53
MSE	16.25		11.44				8.74		9.5		10.22
MAPE	7.49		5.64				5.99		6.06		6.07

Common Nonlinear Trends



Linear Trend Equation

$$F_t = a + bt$$



- F_t = Forecast for period t
- t = Specified number of time periods
- a = Value of F_t at $t = 0$
- b = Slope of the line

Calculating a and b

$$b = \frac{n \sum (ty) - \sum t \sum y}{n \sum t^2 - (\sum t)^2}$$

$$a = \frac{\sum y - b \sum t}{n}$$

Linear Trend Equation Example

t Week	t^2	y Sales	ty
1	1	150	150
2	4	157	314
3	9	162	486
4	16	166	664
5	25	177	885
$\Sigma t = 15$ $(\Sigma t)^2 = 225$	$\Sigma t^2 = 55$	$\Sigma y = 812$	$\Sigma ty = 2499$

Linear Trend Calculation

$$b = \frac{5(2499) - 15(812)}{5(55) - 225} = \frac{12495 - 12180}{275 - 225} = 6.3$$

$$a = \frac{812 - 6.3(15)}{5} = 143.5$$

$$y = 143.5 + 6.3t$$

Associative Forecasting

- Predictor variables - used to predict values of variable interest. a. k. a independent variable
- Usually, demand is the predicted variable (a.k.a. dependent variable)
- Regression - technique for fitting a line to a set of points
- Least squares line - minimizes sum of squared deviations around the line

Regression Methods

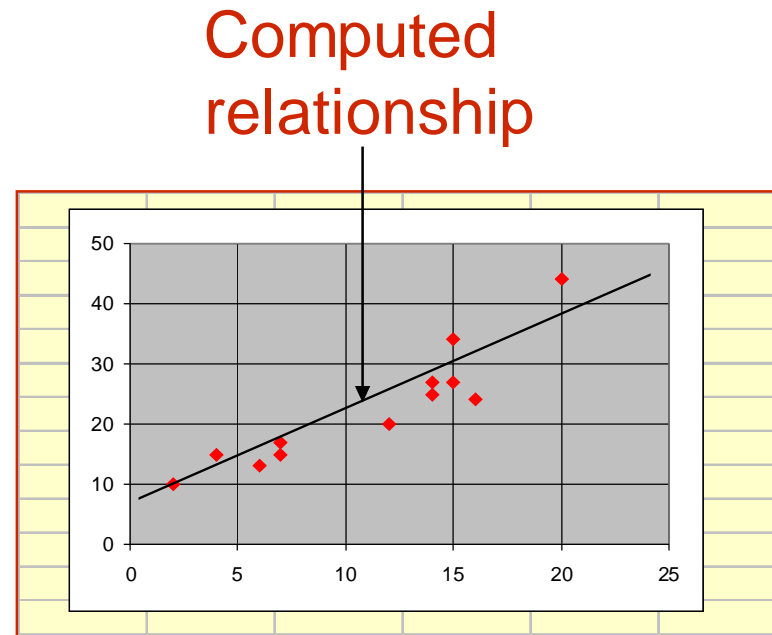
- Linear regression
 - mathematical technique that relates a dependent variable to an independent variable in the form of a linear equation
 - Primary method for Associative Forecasting
- Correlation
 - a measure of the strength of the relationship between independent and dependent variables

Linear Regression Assumptions

- Variations around the line are random
- Deviations around the line normally distributed
- Predictions are being made only within the range of observed values
- For best results:
 - Always plot the data to verify linearity
 - Check for data being time-dependent
 - Small correlation may imply that other variables are important

Linear Model Seems Reasonable

X	Y
7	15
2	10
6	13
4	15
14	25
15	27
16	24
12	20
14	27
20	44
15	34
7	17



A straight line is fitted to a set of sample points.

Linear Regression Model

$$y = a + bx$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{\sum y - b \sum x}{n}$$

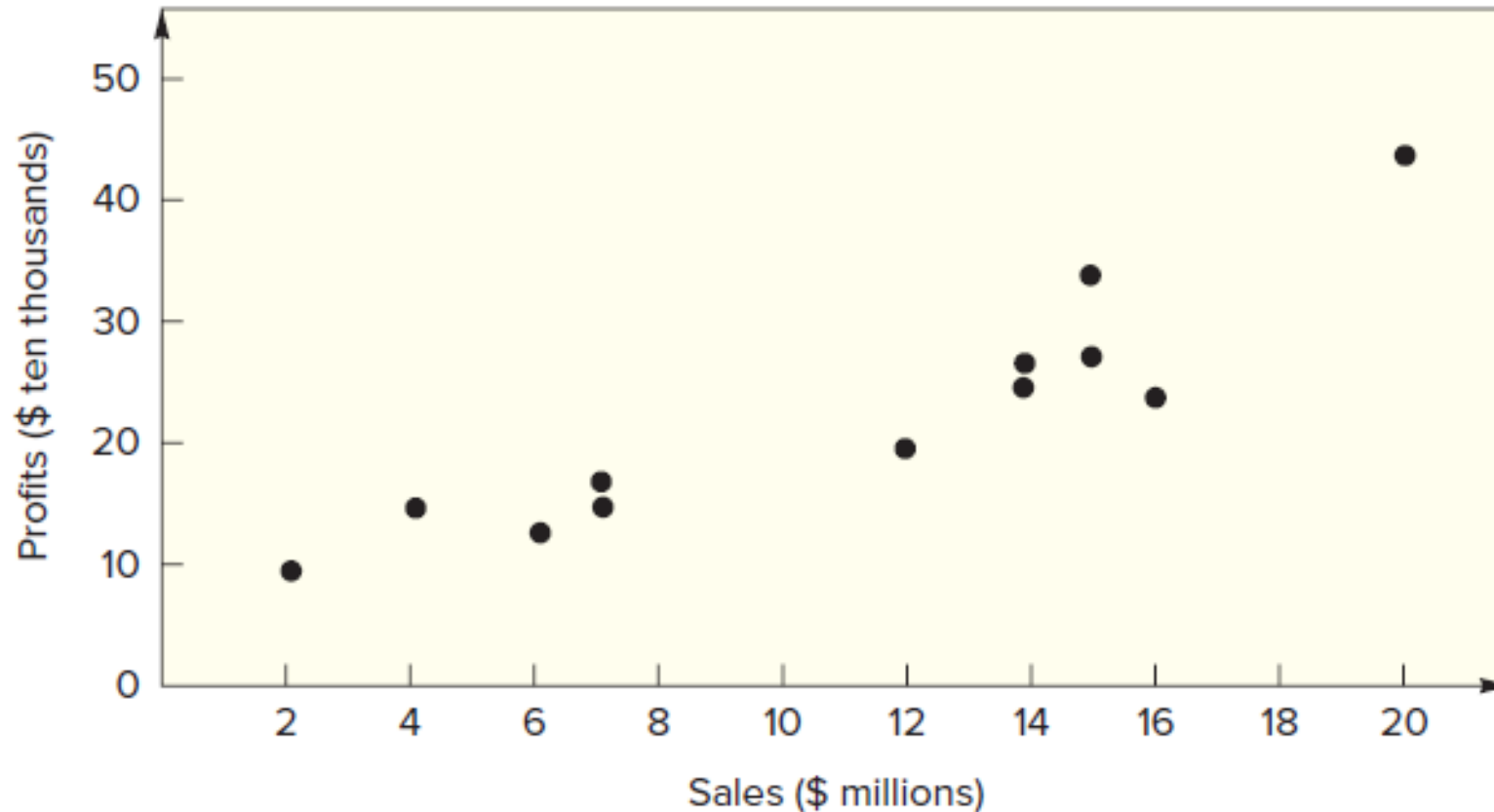
$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \sqrt{n(\sum y^2) - (\sum y)^2}}$$

Linear Regression Example

Healthy Hamburgers has a chain of 12 stores in northern Illinois. Sales figures and profits for the stores are given in the following table. Obtain a regression equation for the data, and predict profit for a store assuming sales of \$10 million.

Unit Sales, x (in \$ millions)	Profits, y (in \$ millions)
\$7	\$0.15
2	0.10
6	0.13
4	0.15
14	0.25
15	0.27
16	0.24
12	0.20
14	0.27
20	0.44
15	0.34
7	0.17

Linear Regression Example



Note: we can reasonably fit a straight line though the scatterplot of the paired data

Linear Regression Model

$$n = 12 \quad \sum x = 132 \quad \sum y = 2.71 \quad \sum xy = 35.29 \quad \sum x^2 = 1796$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{12(35.29) - (132)(2.71)}{12(1796) - (132)^2}$$

$$a = \frac{\sum y - b \sum x}{n} = \frac{2.71 - (0.01593)(132)}{12}$$

$$y = 0.0506 + 0.01593x$$

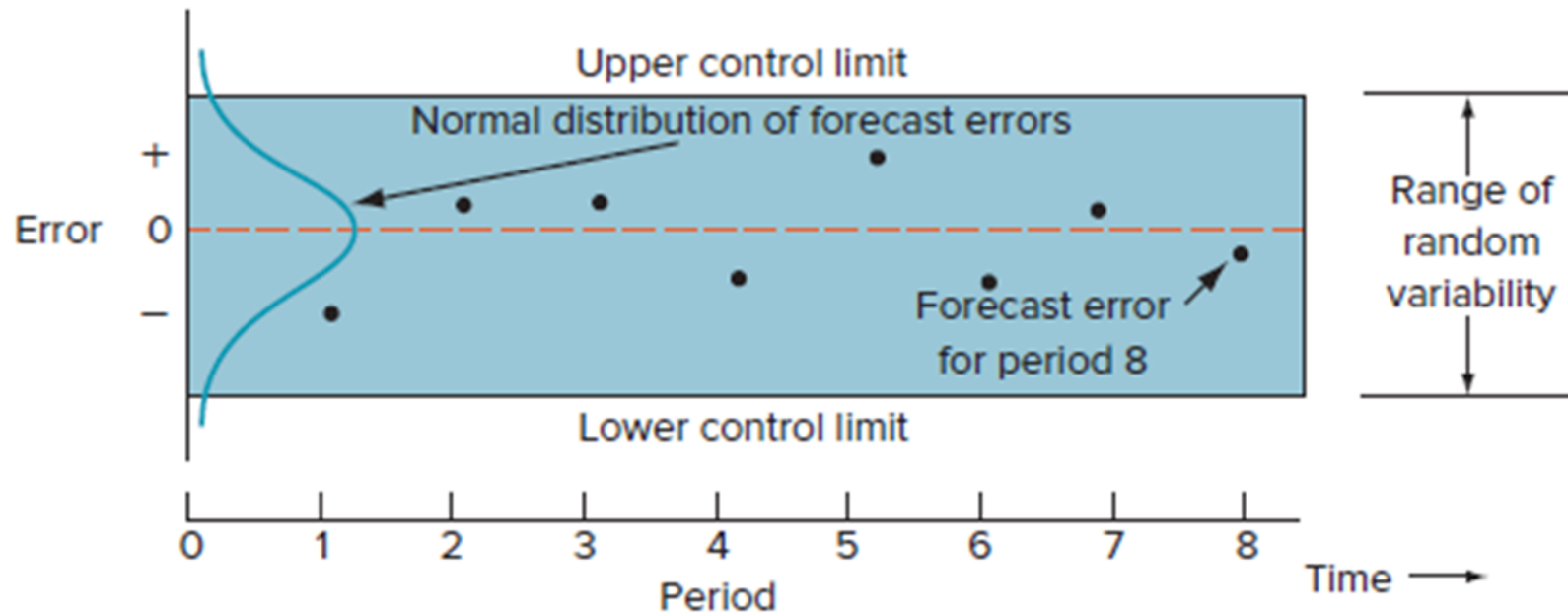
Correlation and Coefficient of Determination

- Correlation, r
 - Measure of strength of relationship between the dependent variable (demand) and the independent variable
 - Varies between -1.00 and +1.00
- Coefficient of determination, r^2
 - Percentage of variation in dependent variable resulting from changes in the independent variable
 - $r^2 = 0.8403$
 - $r = 0.9167$

Controlling the Forecast: Control Charts

- A visual tool for monitoring forecast errors
- Used to detect non-randomness in errors
- Forecasting errors are in control if
 - All errors are within the control limits
 - No patterns, such as trends or cycles, are present

Controlling the Forecast: Control Charts



Controlling the Forecast: Control Charts

1. Calculate the error(s)
2. Calculate the MSE
3. Calculate the UCL and LCL
4. Plot the errors
5. Are the errors within the UCL and LCL?
6. Is there a visible trend or pattern or bias?

$$UCL = 0 + z\sqrt{MSE}$$

$$LCL = 0 - z\sqrt{MSE}$$

Controlling the Forecast: Tracking Signal

- Tracking signal (“TS”)

– Ratio of cumulative error to MAD

$$\text{Tracking signal} = \frac{\sum(\text{Actual-forecast})}{\text{MAD}}$$

Bias – Persistent tendency for forecasts to be Greater or less than actual values.

Question – are the TS values within your limits?

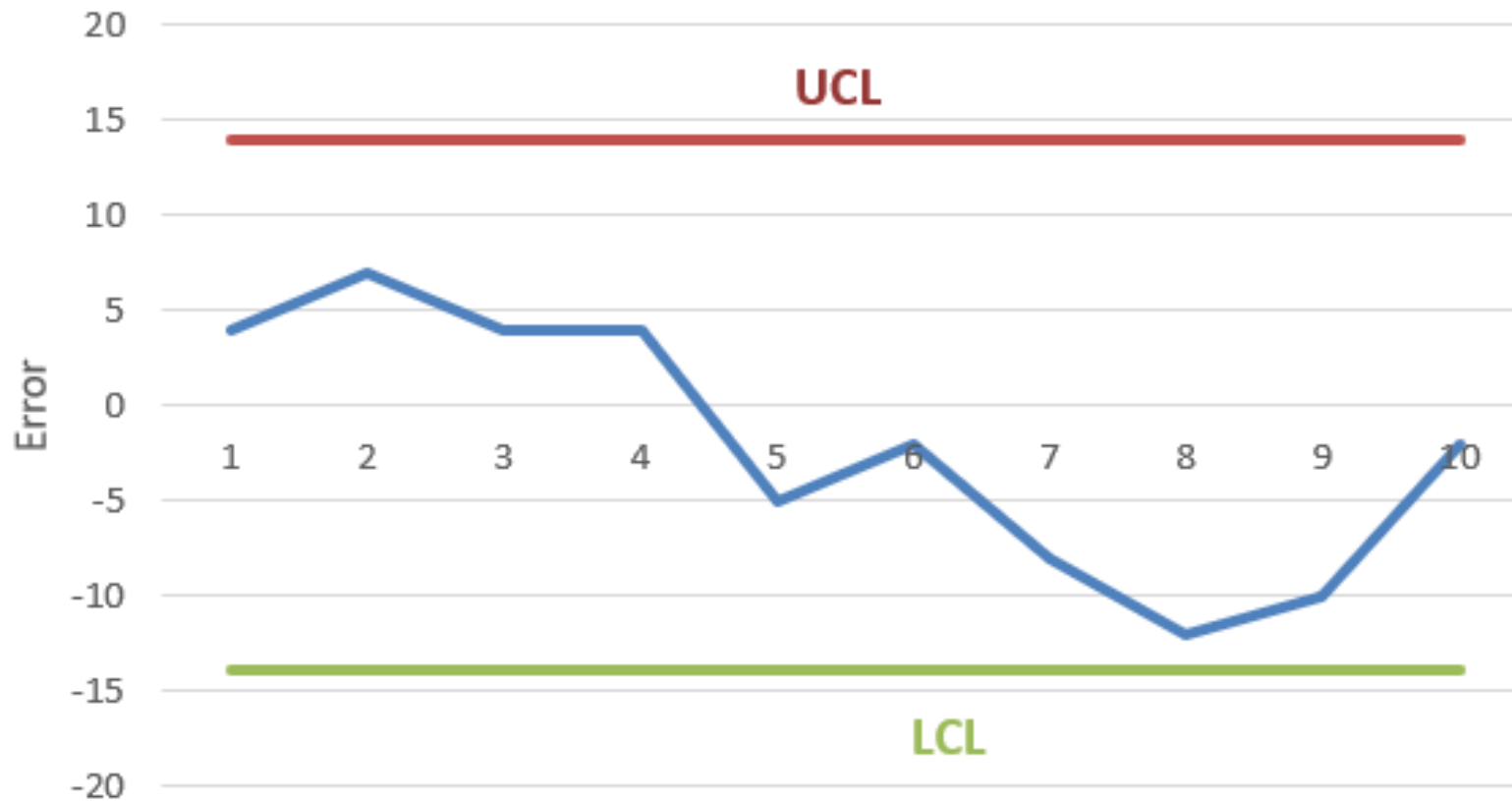
Stevenson Ch. 3: Simplified Example 12

Monthly attendance at financial planning seminars for the past 10 months, and forecasts and errors for those months, are shown in the following table. Determine if the forecast is working using these approaches:

1. A control chart with 2s limits.
2. Tracking signals. Use limits of ± 4 .

Month	<i>A</i> (Attendance)	<i>F</i> (Forecast)
1	47	43
2	51	44
3	54	50
4	55	51
5	49	54
6	46	48
7	38	46
8	32	44
9	25	35
10	24	26

Revised Example 3-12: $\pm 2s$ Control Chart



1. All errors are within the limits
2. There is [apparently] no visible trends, patterns, or biases.
3. Errors are in control and **appears** to be random.

Month	TS
1	1
2	2
3	3
4	4
5	2.92
6	2.77
7	0.82
8	-1.39
9	-2.89
10	-3.45

- All TS values are within specified ± 4 limit.

Conclusion: Given the combination of the Control Chart and the TS values, the forecast is acceptable.

Sources of Forecast errors

- Model may be inadequate
- Irregular variations
- Incorrect use of forecasting technique

Choosing a Forecasting Technique

- No single technique works in every situation
- Two most important factors
 - Cost
 - Accuracy
- Other factors include the availability of:
 - Historical data
 - Computers
 - Time needed to gather and analyze the data
 - Forecast horizon

A professional's opinion

“To put it another way, the easy part of forecasting is fitting a model and the hard part is knowing when the best fit on past data will yield a worse forecast on unknown data,…”

- *Nate Silver, Editor-in-Chief, FiveThirtyEight.*

Source: https://nymag.com/intelligencer/2020/10/nate-silver-and-g-elliott-morris-are-fighting-on-twitter.html?utm_source=Sailthru&utm_medium=email&utm_campaign=Intelligencer%20-%20October%202015%2C%202020&utm_term=Subscription%20List%20-%20Daily%20Intelligencer%20%281%20Year%29

Operations Strategy

- Forecasts are the basis for many decisions
- Work to improve short-term forecasts
- Accurate short-term forecasts improve
 - Profits
 - Lower inventory levels
 - Reduce inventory shortages
 - Improve customer service levels
 - Enhance forecasting credibility

Supply Chain Forecasts

- Sharing forecasts with supply can
 - Improve forecast quality in the supply chain
 - Lower costs
 - Shorter lead times